Rotational Kinetic Energy

Please Circle Your Lab day: M T W T F

Name: ________________________

Partner: ________________________

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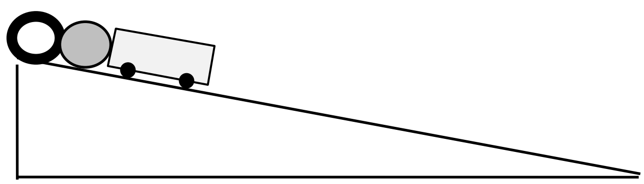
PURPOSE: To thoroughly study the motion of objects rolling down an incline. This involves developing a theoretical relationship between the speed of an object after traveling down the incline and properties of the object and the incline.

- **Project #1**: Which rolling object is going the fastest at the bottom on an incline?
- **Project #2**: What is a theoretical prediction for the speed of an object rolling down an incline?
- **Project #3**: How well do theory and experimental observation compare?

Project #1: Which is going the fastest at the bottom of the ramp: the cart, the hollow cylinder, or the solid cylinder?



1. Put the cart, solid cylinder, and hollow cylinder all in a row at the top of the ramp. Let them all roll down the ramp together (starting from rest). Describe your observations.

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2. What happens if you change the order (for example: hollow cylinder first, solid cylinder, then cart)? (In HW1 you will do calculations which show why these cases are different.)

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3. Which object is going fastest at the bottom of the ramp when they are rolled separately?

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4. Rank each of the properties in the following list by what you predict is their importance in determining the speed of an object rolling down an incline. (1 = most important, 2 = next most important, etc.). Note that the cart has no radius.

weight	____________	radius	____________
mass	____________	distribution of mass	____________

Explain why you believe your ranking is correct.

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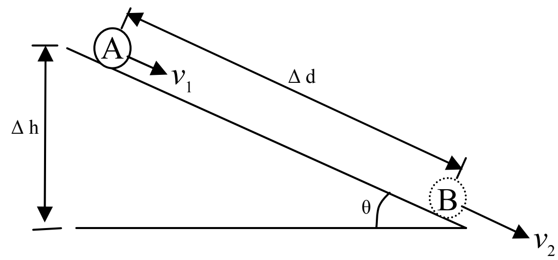
Project #2: What is a theoretical prediction for the speed of an object rolling down an incline?

THEORETICAL ANALYSIS: Apply the law of conservation of energy to an object rolling down an incline. In the following, use the conventional symbols to answer each question with a mathematical expression. We will begin with the simplest situation: $\boldsymbol{v_1} = 0$ and no retarding forces.



1. What is the change of gravitational potential energy of the object after it goes from position ***A*** relative to position ***B***? Write down a formula.

2. What is the change of total kinetic energy of the object from position ***A*** to position ***B***? (Hint: be careful! Don't forget rotational KE.)

3. Assuming that retarding forces can be neglected, apply the law of conservation of mechanical energy to derive an expression for the speed of the object when it passes ***B*** in terms of the vertical distance it traveled, Δh, going from ***A*** to ***B***. Since the moment of inertia of all these objects is proportional to mr^2, use $I = fmr^2$ in your derivation, where f is the proportionality constant determined by the object's shape. ($f = 0$ for the cart, because it does not rotate. But still include f in your algebraic derivation.)

4. What does theory say are the most important properties of the object that determine its speed at ***B***?
Again rank each of the following properties in terms of their importance in determining the speed of an object rolling down an incline. (1 = most important, 2 = next most important, etc.):

weight	____________	distribution of mass	____________
mass	____________	other (name it)	____________
radius	____________		

If you changed your rankings from Step 1, **explain** why you believe your new ranking is correct.

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PROJECT #3: How well do theory and experimental observation compare?

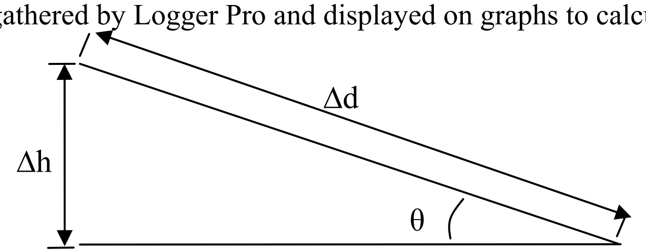
1. The theoretical analysis based on the **changes** in PE_{grav} and KE shows that after traveling a distance, $\boldsymbol{\Delta d}$ (= $\boldsymbol{d_2 - d_1}$), along the ramp the speed of an object is related to the **change** in vertical height, $\boldsymbol{\Delta h}$ (= $\boldsymbol{h_2 - h_1}$), through which it traveled.
Theory can be tested by measuring the vertical distance traveled, calculating a theoretical final speed, and comparing it to the measured final speed. It is not experimentally convenient to have an initial speed, $\boldsymbol{v_1} = 0$. It is more convenient to start collecting data and release the object. Then examine the resulting graphs, to find a region that consists of reliable data. This region might not include the moment when speed was zero.

2. Now repeat the theoretical analysis for the case when the initial velocity $v_1 \neq 0$. Insert into the equation below, the expressions for the KE and PE at points ***A*** and ***B***

.

$$\frac{\Delta KE}{m} = -\frac{\Delta PE}{m}$$

Notice that if you have used $I = fmr^2$, then m cancels out, so that you obtain a relationship between $\boldsymbol{\Delta v^2}$ and $\boldsymbol{\Delta h}$. Thus theory can be tested by determining a $\boldsymbol{\Delta h}$ that corresponds to a $\boldsymbol{\Delta v^2}$.
Since the motion detector records distance and since there is a geometrical relationship between $\boldsymbol{h}$ and $\boldsymbol{d}$, you can use data gathered by Logger Pro and displayed on graphs to calculate $\boldsymbol{\Delta v^2}$ and $\boldsymbol{\Delta h}$.



Examine this Figure. Measure Δh and Δd. (Δd is the length of the track, and Δh is height difference of the track's ends.) Find the relationship between Δh and Δd. Recall similar triangles from 8th grade.

3. Examine the following experimental procedure to test the theory.
Set up the track as an incline, put the motion detector near the higher end, turn on the ULI box, run Logger Pro, select a "Graph Layout" of "3 Panes" (distance, velocity, & acceleration), place an object in front of the motion detector, click on "Collect", release the object, examine the graphs to locate a section of usable data (hint: look at the acceleration graph), use "Examine" to obtain values for $\boldsymbol{d_1}$, $\boldsymbol{d_2}$, $\boldsymbol{v_1}$, and $\boldsymbol{v_2}$ at the times $\boldsymbol{t_1}$ and $\boldsymbol{t_2}$, and finally determine $\boldsymbol{g\,\Delta h}$ and $\boldsymbol{(1/2)(v_2^2 - v_1^2)(1+f)}$.

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4. Use the table below or create your own table (for example, in Excel) to record all the necessary measurements to test the theory. Perform the experiment for each of the objects. You should make three "runs" to be sure to collect good quality data.

t_1	d_1	v_1	t_2	d_2	v_2	Δh	$\Delta(v^2)$	$g\Delta h$	½ $\Delta(v^2)$ $(1+f)$	% diff

5. Are the percent differences between $\boldsymbol{g\,\Delta h}$ and $\mathbf{(1/2)}(v_2^2 - v_1^2)\mathbf{(1 + \boldsymbol{f})}$ for each object reasonable? Explain why you think so.

6. In **Project #2**, **Step 3.**, frictional effects were ignored. Modify the analysis of **Step 3.** to take into account the energy lost to retarding forces. Assume that the effects of retarding forces can be treated as an *effective frictional force*. Derive a mathematical expression for the *effective* coefficient for friction, $\boldsymbol{\mu}$, and solve for it in terms of the height, angle θ, and the speed of the object at position ***B***. (Hint: use the work-energy principle.)

Determine your experimental value of $\boldsymbol{\mu}$ for each object.

Cart _______ Solid Cylinder ______ Hollow Cylinder _______

7. Are these values for $\boldsymbol{\mu}$ reasonable? **Explain** why you think so?

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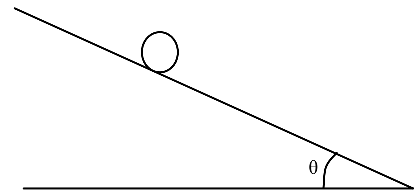
Homework:

H1. Indicate on the drawing all the forces that are acting on the object as it is rolling down the incline. (**Note:** assume that the object is not sliding or slipping on the surface of the incline.)



H2. Draw the +x-axis parallel to the incline and the +y-axis perpendicular and upward. Write down Newton's second law for the x- and y-directions and for rotation. Include all the forces. (**Hint:** Assume the object is rolling without sliding. Therefore the force of friction is what is making the object rotate. This is not the same force of friction involved in **Project #3, Steps 6 – 7**.)

H3. Solve the above equations for the translational acceleration of the object.

H4. Use this expression for the acceleration and derive an expression for the speed of the object after going from position ***A*** to position ***B*** in the figure on page 2.

H5: How does this expression compare with that derived in **Project #2, Step 3**?