A Special Relativity Kinematics Problem: "The Rocket and Hangar"

1. One of the most surprising consequences of special relativity is that sequences of events may be seen very differently from different frames of reference. Imagine a car that is 50 meters long driving through a garage that is 100 m from front to back. The car approaches the closed front of the garage and someone opens the front door. The car passes through the garage, the rear door is opened and the car passes out of the garage. This sequence could be described as follows:

<table>
<thead>
<tr>
<th>Event Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Front bumper of car reaches front door; front door is opened</td>
</tr>
<tr>
<td>2. Rear bumper of car reaches front door; front door is closed</td>
</tr>
<tr>
<td>3. Front bumper reaches rear door; rear door is opened</td>
</tr>
</tbody>
</table>

Now image a 100 m rocket traveling at 0.6c toward a hanger that is 90 m from front to back. Two people -- an observer (H) at rest with respect to the hangar and the captain (R) at rest in the rocket -- will both observe the rocket pass through the hangar.

The observer H tells the captain R that (due to relativistic length contraction) he can close the front and rear doors of the hangar and the rocket will be briefly enclosed within the hangar. He will open the rear door just as the nose reaches it, and the rocket will pass through the hanger.

Now consider this from the standpoint of the captain of the rocket ship. Will he see the rocket crash through either the front door or the rear door of the hangar? Since the hangar is moving toward him at 0.6c its length will be contracted to become even shorter than its 90m rest length.

Recall that an event is described by its position and time coordinates. For our problem, we will need only one space dimension for the position and one time as the coordinates: \( x_{H}, t_{H} \) in the hangar frame, and \( x_{R}, t_{R} \) in the rocket frame. The origin \( x_{H} = 0 \) is at the front door in the hangar frame, and the other origin \( x_{R} = 0 \) is at the nose in the rocket frame. The clocks are synchronized, \( t_{H} = 0 = t_{R} \) just as the nose of the rocket ship enters the front door of the hangar (the two space origins are coincident at that instant). As you answer the following questions you should be able to explain the strange situation in which a long rocket is enclosed within the shorter hangar.

1. What is the formula for relativistic length contraction? Explain each of the symbols used.

2. How long is the moving rocket according to H? Would H say that the rocket could be enclosed within the hangar (i.e. both front and rear doors closed simultaneously)?
3. How long is the moving hangar according to R?

In the following questions you will be filling in portions of the table on page 3. Remember that the distance traveled at constant speed is given by $\Delta x = v \Delta t$, as long as the quantities $\Delta x$, $\Delta t$, and $v$ are all measured with respect to the same coordinate system. Clearly show your calculations.

4. Find the time H says the tail of the rocket arrives at the front door, i.e. find the time coordinate $t_H$ for event $E_2$.

5. Find the time of this same event in R's frame of reference. (Hint: how far must the front door travel in R's frame from the nose of the rocket to the tail; how long does this take?)

6. Find the time in H's frame for the nose of the rocket to reach the rear door, i.e. find the time coordinate $t_H$ for event $E_3$. (Hint: how far is it from the front to the rear door in the hanger frame; how long will it take the nose of the rocket to travel this distance?)

7. Find the time in R's frame for event $E_3$. (Hint: How far was the rear door of the hangar from the
Here is a table of events with some of the most obvious space and time coordinates entered. Be sure to ask if you do not understand any of these entries. Fill in the table using your answers to questions 2 – 7).

<table>
<thead>
<tr>
<th>EVENT</th>
<th>H's (X_h,t_h)</th>
<th>R's (X_r,t_r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_1 nose of rocket at front door</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>E_2 tail of rocket at front door</td>
<td>(0, _________)</td>
<td>(-100m, _________)</td>
</tr>
<tr>
<td>E_3 nose of rocket at rear door</td>
<td>(90m, _________)</td>
<td>(0, _________)</td>
</tr>
</tbody>
</table>

8. List the events E_1, E_2, E_3, in the sequence (time order) that they occurred in the hangar frame. Then list the same events in the sequence they occurred in the rocket frame.

9. Write one or two complete sentences describing the sequence of events as seen by H.

10. Write one or two complete sentences describing the sequence of events as seen by R. This should clearly explain how the rocket ship passed through the hangar, although it is too long in this frame of reference to fit inside the hangar.
II. A second surprising consequence of Special Relativity is the equivalence of mass and energy that is described by Einstein’s well-known equation

\[ E = mc^2 \]

Use the equivalence of mass and energy to answer the following questions.

A. The Sun’s luminosity is \(3.8 \times 10^{26}\) joules/second. How many tons of matter destroyed each second to produce this energy? You may assume that 1 ton = 907 kg.

B. We measure nuclear explosives in terms of kilotons of TNT (the bomb that destroyed Hiroshima in WWII was a 20 kiloton weapon). 1 kiloton = \(4.19 \times 10^{12}\) Joules. How many Hiroshima-type bombs would you have to explode every second to equal the Sun’s energy output?

C. EXTRA CREDIT. If a star converted every bit of its mass into energy the conversion efficiency would be 100%. However, the Sun converts hydrogen into helium with an efficiency of only 0.7%. The Sun has a mass of \(2 \times 10^{30}\) kg. How long will the Sun live if it converts all of its hydrogen into helium?